

IAS

INSTITUTE FOR  
ADVANCED STUDY

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# 2024 Marston Morse Lecture Series

## Minimal Surfaces and the Isoperimetric Inequality

The isoperimetric inequality has a long history in the geometry. In this lecture, we will discuss how the isoperimetric inequality can be generalized to submanifolds in Euclidean space. As a special case, we obtain a sharp isoperimetric inequality for minimal submanifolds of codimension at most 2, thereby answering a question going back to work of Carleman. The proof of that inequality is inspired by, but does not actually use, optimal transport.



**Monday  
February 26th**

Simonyi 101 and Zoom

## Singularity Models in 3D Ricci Flow

The Ricci flow is a natural evolution equation for Riemannian metrics on a given manifold. From the point of view of PDE, the Ricci flow is a system of linear parabolic equations, which can be viewed as the heat equation analogue of the Einstein equations in general relativity. The central problem is to understand singularity formation. In other words, what does the geometry look like at points where the curvature is large? In his spectacular 2002 breakthrough, Perelman achieved a qualitative understanding of singularity formation in dimension 3; this is sufficient for topological conclusions. In this lecture, we will discuss more recent developments which have led to a complete classification of singularity models in dimension 3.



**Wednesday  
February 28th**

Simonyi 101 and Zoom

## Scalar Curvature Rigidity of Polytopes

A central theme in differential geometry involves studying Riemannian metrics satisfying various curvature positivity conditions. The weakest condition one can impose is the positivity of the scalar curvature. Inspired by Toponogov's triangle comparison theorem on manifolds with nonnegative sectional curvature, Gromov has proposed a comparison principle for polytopes carrying metrics with nonnegative scalar curvature. In this lecture, we will discuss a recent result which verifies Gromov's conjecture under a stronger hypothesis. The proof uses the Fredholm theory for Dirac operators on manifolds with boundary, as well as an estimate from harmonic analysis due to Fefferman and Phong.



**Friday  
March 1st**

Simonyi 101 and Zoom



**Zoom**

[https://theias.zoom.us/j/87654960483?](https://theias.zoom.us/j/87654960483?pwd=dWhpQnBjKzJZaFFKQXI4YkJjUGJBZz09)  
[pwd=dWhpQnBjKzJZaFFKQXI4YkJjUGJBZz09](https://theias.zoom.us/j/87654960483?pwd=dWhpQnBjKzJZaFFKQXI4YkJjUGJBZz09)

Meeting ID: 876 5496 0483  
Passcode: 3267